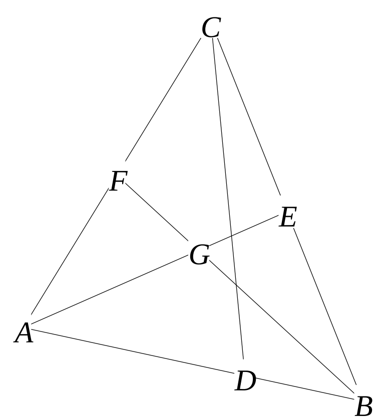
Introductory Activities

0. Start *Wingeom* by double-clicking its icon. Click the **2-dim** item on the main menu bar. This will create a small drawing window. From now on, you will be using its menu bar. Items in boldface are menu items or dialog buttons that are meant to be clicked with the mouse. Keyboard input, such as *Enter*, *Escape*, and *Ctrl+W*, is generally written in italics.

1. The left and right mouse buttons perform different functions, depending on the item that is checked on the **Btns** menu. The primary drawing mode is **Btns|Segments**. Check this item by clicking it, then point at three random places in the drawing window and click the <u>right</u> button. This labels three points *A*, *B*, and *C*. Point at *A*, hold down the <u>left</u> button, slide the pointer to *B*, and release the button. Segment *AB* now appears on the screen (unless the button release was not close enough to *B*). Repeat the process to draw segments *BC* and *CA*.

2. Right-click a point on segment *AB*. Its assigned label will be *D*, because the program always selects the first available label for a new point. Right-clicking is one way of marking a point on a segment. Here is another: Click **Point|on Segment**, type the list *BC*,*CA* into the "relative to segment" box, notice that the "coordinate" box shows the value 0.5, and click **mark**. Labels *E* and *F* appear at the midpoints of segment *BC* and *CA*, respectively. Close this dialog box by pressing *Escape* or by clicking **close**. Use the left button to draw segments *CD*, *AE*, and *BF*, as in step 1. Point at the intersection of *AE* and *BF* and click the right button. The intersection point should now be labeled *G*, as shown.



3. Click **Btns|Drag vertices** to put the mouse into a different mode. Point at *B*, hold down the left button (notice that the label color changes), and slide the mouse. Point *B* moves, while *everything else in the figure adjusts its position accordingly*. In particular, *E* maintains its status as the midpoint of segment *BC*, and *D* holds its relative position on segment *AB*. These adjustments continue until you release the button. This is called *dragging* point *B*. Points *A* and *C* can also be dragged. If you try to drag either *E* or *F*, however, the whole triangle moves rigidly. Think about why this happens. If you try to drag *D*, you will find that it *does* move — but only along the segment it was placed on. Notice that the intersection *G* of the medians *AE* and *BF* is not usually on segment *CD*.

4. Click **Meas** to open a new dialog box. The cursor is blinking in an edit box. Type the ratio *AD/AB* into the box (the program does not distinguish between upper and lower case letters, by the way) and press *Enter*. The current value of this ratio is shown in both the dialog box *and* the drawing window. This is because the measurement dialog *must* be closed (just press *Escape*) before drawing operations can resume. Return to dragging point *D* along *AB*. The displayed value of *AD/AB* tells you the exact position of *D* on the segment. Notice the value of this ratio when *G* seems to be on segment *CD*. Is it what you would expect?

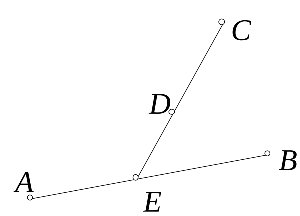
5. Not only does the program assign labels to new points, it also centers the labels directly over the points, and it puts measurements in the upper left corner of the drawing window. You can override these choices. First click **Btns|Text** to put the mouse into a new mode. Point at label *A* and press the *right* button. A new dialog box allows to change the label. Type *P* into the box and press *Enter*. You will see label *A* change to *P*. In the same way, change label *B* to *Q*. Notice that the displayed ratio *AD/AB* is now “undefined”, so click **Meas**, select *AD/AB* in the list box, and click **delete**. Type *PD/PQ*, press *Enter*, and press *Escape*. Next point at any label in the figure, hold down the *left* button, slide the mouse a short distance, and release the button. The point does not move but the label does. Notice that the point is marked by a small circle, usually hidden by the label. Click **Edit|Labels|Offset** to move all the labels (this seldom places every label satisfactorily, however). Click **Edit|Labels|Home** to recenter all the labels. To choose a different font for the labels, click **Edit|Labels|Font**.

6. When the mouse is in **Btns|Text** mode, the right button is used to place *new* text or edit *old* text anywhere on the screen, and the left button is used to drag (reposition) text. For example, drag the measurement to a spot that is closer to segment *PQ*. To send all calculations back to their default positions, click **Other|Measurements|Home**.

7. Click **File|New** to start fresh. The program asks if you want to save your work. Respond **no**.

8. Put the mouse into **Btns|Segments** mode, right-click four new points, then use the left button to create segments *AB* and *CD*. Put the mouse into **Btns|Drag vertices** mode, and drag some of the points so that the segments intersect. Here is a new way of labeling the intersection that is occasionally useful: Click **Point|Intersection|Line-line**, type *AB* into one edit box, *CD* into the other, and press *Enter*. Label *E* should appear. Press *Escape* to close the box.

9. Make sure that **Other|Autoextend** does not have a check mark. (If necessary, click it to remove the check.) Drag *B* somewhere so that segments *AB* and *CD* no longer intersect, and watch *E* disappear. Now activate the **Other|Autoextend** feature by clicking it. Notice that the segments are now extended as much as necessary to show their intersection, regardless of where the points are dragged.



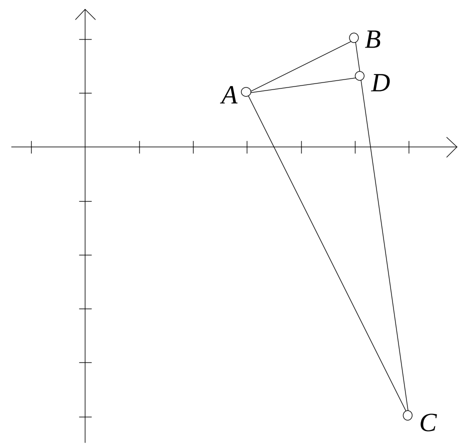
10. Click **Edit|Undo** (or press *Ctrl+Z*). This undoes the most recent construction step, so intersection *E* is no longer labeled. Click **Edit|Undo undo** (or press *Ctrl+Y*) to redo the last step. You can always click **Other|Lists|History** to see a step-by-step description of the current figure. (The text display does not change if you do anything to the drawing, however. To update the contents, it is necessary to click **close** and then re-open the window.)

11. Make a fresh start by clicking **File|New**, then click **Point|Coordinates** to open a coordinate-entry dialog box. Type 3 into the *x* box, 1 into the *y* box, and press *Enter* (or click **mark**). Point *A* = (3, 1) should appear. In the same way, label the points *B* = (5, 2) and *C* = (6, –5). Close this dialog box. Click **View|Axes** (or press *Ctrl+A*) to make the coordinate axes disappear. (You can make them reappear in the same way.)

12. Put the mouse into **Btns|Drag vertices** mode, then discover that you can not drag *any* of the points without moving the whole sheet of graph paper. This is because the points were defined to keep them in one place.

13. Click open the **Meas** dialog box. Type <*ABC* into the edit box and press *Enter*. The size of angle *ABC* is displayed. The inequality symbol tells the program that an angle size is requested. If you try to describe the same angle as just <*B*, the program will not understand, for it wants a three-label description. Type *ABC* into the edit box and press *Enter* to see the area of triangle *ABC*. Type *AB*+*BC*+*CA* and press *Enter* to see the perimeter of triangle *ABC*. To deselect the highlighted perimeter in the dialog's list box, click it. Then click the angle measurement to select it. Clicking the **hide** button makes the measurement disappear from the screen, and clicking **show** makes it reappear. Press *Escape* to close the dialog box.

14. If you want dimensional information appended to displayed measurements (to designate length, area, and angles), the item **Other|Measurements|Units** must be checked. Until the screen is forced to refresh itself, nothing happens after you change the status of this item.



15. Click **Line|Perpendiculars|Altitude**, type *BC* into the "perp to" box, *A* into the "from pt" box, and click **draw**. Segment *AD* appears, constructed so that *D* is on segment *BC* and angle *ADB* is a right angle. Press *Escape* to close the dialog box.

16. Click open the **Meas** dialog box again. Type *D* and press *Enter*. You should see that the coordinates of *D* are (5.1, 1.3). To confirm that *AD* does intersect *BC* perpendicularly, type <*ADB* and press *Enter*. Press *Escape* to close the dialog. Click **Other|Lists|Lines** to open a text window that enumerates all combinations of collinear points in the figure. If the coordinate axes are showing, a Cartesian equation is displayed for each line, as well as its slope. Click **close** to hide this text window.

17. If you would prefer to have the mouse functions more visible (instead of hidden in a menu), click **Btns|Toolbar**. This small, movable dialog box displays the current mouse function, and it gives you another way to alter it. Put the mouse into **Drag vertices** mode. As you noticed earlier, dragging any of the labeled points simply slides the sheet of graph paper across the screen. Do so now, and try to make some of its vertices disappear from view (if your drawing window does not fill the screen, you can make the entire triangle disappear). To quickly restore any figure to the center of the screen, so that all of its parts are visible, click **View|Window** (or press *Ctrl+W*). This also repositions any measurements.

18. Click **File|New** to begin a new figure. If the coordinate axes are still showing, press *Ctrl*+*A* to turn them off. Click **Units|Polygon|Parallelogram**, type 12 into the first "side" box, change the "angle" to 90 and the second "side" to 4, then press *Enter* (or click **ok**). A rectangle *ABCD* should appear. (Because it is three times as wide as it is tall, it might not fit well in the

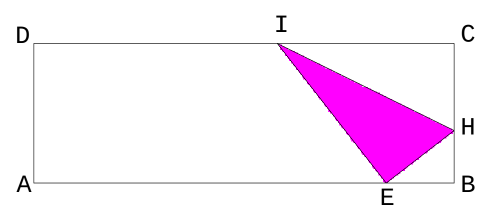
current window, but you can change the size and shape of a window by dragging its border.) Put the mouse into **Btns|Segments** mode and right-click a point *E* onto the segment *AB*, closer to *B* than *C* is. Then use the left button to draw the segment *CE*.

19. Click **Line|Perpendiculars|Bisector**, type *CE* into the edit box, and press *Enter*. The bisector *FG* goes through the midpoint *F* of segment *CE*, and it should also intersect sides *BC* and *CD*. Use the right button to label these intersections, with *H* on *BC* and *I* on *CD*. Then use the left button to draw segments *EH* and *EI*.

20. Time to clean up. Points *F* and *G* are no longer needed, so click **Edit|Delete|Point**, type *FG* into the box, and press *Enter*. Click **Edit|Delete|Line**, type the list *CE*,*CH*,*CI* into the box, and press *Enter*. To turn the bisector *HI* into a segment, click **Line|Extensions**, type the list *HI*,*IH* into the box, and press *Enter*. (The rays *HI* and *IH* are thereby turned off. Rays can be turned on in the same fashion.)

21. Use the left button to reconnect segments *DI*, *IC*, *CH*, and *HB*. The reason for this strange move will be explained next. Click **Edit|Highlights|Line attributes**, type the list *CH*,*CI* into the edit box, click the **style** button until “dotted” appears, and click **apply**. Because the program was told to *forget* that *C*, *I*, and *D* are collinear (this was the reason for erasing segment *CD* and then redrawing it in two pieces), the dotted style does not apply to *DI*. Close the dialog box.. Click **Edit|Highlights|Fill**, type *EHI* into the “polygon” box, click the magenta cell, and press *Enter*. Triangle *EHI* should now be magenta. Close this dialog box, and press *Ctrl+W* to center the drawing, which should now look like the illustration below.

22. This construction is meant to simulate the folding of a rectangular sheet of paper, so that one corner (*C*) is matched with a point (*E*) on another edge. The dotted segments mark where the paper used to be, before it was folded over, and the red color is found on the underside of the sheet.



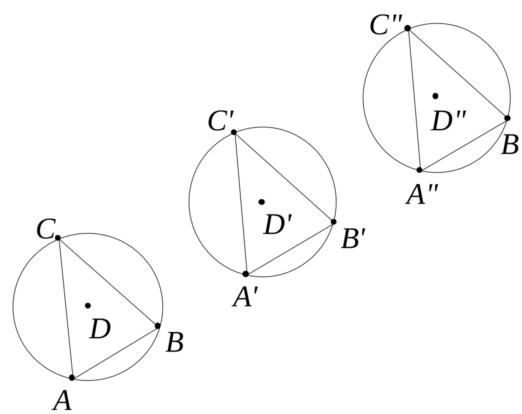
23. Put the mouse into **Btns|drag vertices** mode and make sure that the **Other|Autoextend** item is unchecked. Use the left button to slide *E* along side *AB*. Move the mouse slowly, keeping *E* close to *B*. If you slide *E* too far left, the construction will collapse, because the instructions make no sense if *EB* is greater than *BC*. (*H* has to be on side *BC*.)

24. Open the **Meas** dialog and ask for <*BHE* and 2<*HIC* (one at a time, pressing *Enter* after each). *Escape* from the dialog box. Notice that the equality of these two measurements is unaffected by moving *E* along *AB*. Explain why this should have been expected.

25. Click **File|New** to begin a new figure. (It is possible to have several drawings open at the same time — just click **2-dim** again on the main menu bar — but this is not necessary for this tour.) Click **Units|Random|Triangle**. This is a one-click method of putting three random points on the screen and connecting them with segments.

26. Click **Circle|Circumcircle**. The resulting dialog box probably already has *ABC* highlighted in the edit box, so just press *Enter*. The program constructs the circle that goes through all three vertices of the triangle. The center of the circle is labeled *D*. The program keeps track of each circle by recording its center and at least one point on the circle. Deleting the center of a circle would therefore require that the circle itself be deleted. To safeguard against this happening inadvertently, the program will not let you do it: Click **Edit|Delete|Point**, type *D* into the edit box, and press *Enter*. Click **ok** to close the error-message box. The same message would result if you tried to delete a point that was the only point marked *on* a circle.

27. Click **Transf|Translate**, type 2 into the "multiple" box, and *AB* into the "vector" box (if it is not already there). Leave the "vertices" box alone, for it lists all the vertices currently in the figure, and that is the plan. To see the result of sliding the whole figure twice the length of vector *AB*, press *Enter*. Notice that the new figure consists of two triangles, two circles, and eight vertices, and that the four new vertices have been labeled by attaching primes to the original labels. Click **Transf|Last repeat** (or press *F7*). This applies the current transformation to its most recent images. The figure should now look like the illustration.



28. With the mouse in **Btns|Drag vertices** mode, drag vertex *C* around the screen, and notice what happens to all the images. Because the translation is defined in terms of segment *AB*, the effect is quite different if you drag *B* around the screen instead. Try it. Also notice that you can not drag points other than *A*, *B*, or *C* without dragging the entire figure rigidly. Why is this to be expected?

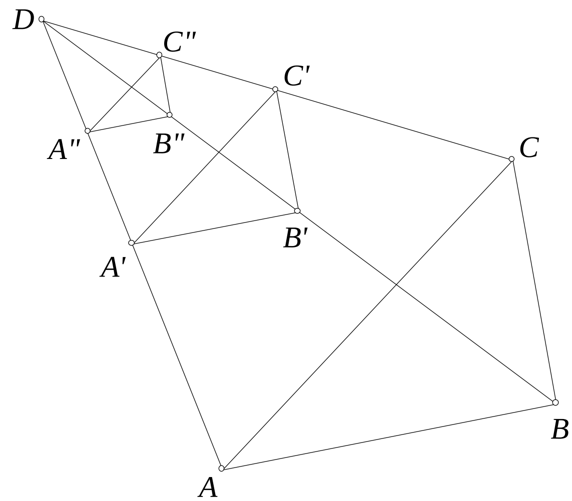
29. Click **Transf|Translation** again, type 2 into the "multiple" box, type *AC* into the "vector" box, and leave the "vertices" box filled with the default list of all vertices. Press *Enter*, then press *F7*. You should now see nine triangles, nine circles and their centers, and thirty-six labels. The program labels transformed vertices by using subscripts and primes. Having this many labels on the screen at the same time can be confusing, so press *Ctrl+L* (which is easier than clicking **Edit|Labels|Hide**) to turn all the labels off. (They can be turned on in the same way.) With the labels off, press *Ctrl+D* (which is the same as clicking **Edit|Labels|Dot mode**) several times. Notice that points are marked by open circles, closed circles, crosses, or nothing at all. With the open circles on the screen, click **Edit|Labels|Bullet size**, type a one-digit number into the box, and press *Enter*. The default size for the circles is 4, but you may prefer a different size (and the program will remember).

30. Click **File|New** to begin a new figure. Click **Units|Random|Triangle** again. With the mouse in **Btns|Segments** mode, right-click a random point *D* onto the screen, well away from triangle *ABC*. Click **Transf|Dilate**. This activates a dialog box that is used to define both rotations and dilations (or combinations of the two). It is now configured for a simple dilation,

which means that 0.0 shows in the "angle" box. Type *D* into the "center" box, *ABC* into the "vertices" box, and press *Enter*. Press *F7* once to apply the dilation again.

31. Use the left button to draw segments *DA*, *DB*, and *DC*. Points A' and A'' will look like they also lie on segment *DA*. Because this property is indeed built into the definition of a dilation, you know that these points *are* collinear. Nevertheless, the points were not marked *on* the segment (which appeared later in the construction, anyway), and the program does not know any theorems of geometry, so it does not recognize that A' and A'' are in fact on *DA*. To see evidence of this, click **Edit|Delete|Line**, type AA' into the box, and press *Enter*. Notice that nothing disappears from the figure.

32. The figure should resemble the illustration. Put the mouse into **Btns|Drag vertices** mode, and drag the primary vertices (namely *A*, *B*, *C*, and *D*) around the screen. The response of the figure when *D* is moved is quite different from the response of the figure when *A* is moved.



33. If the **Transf|Save labels** item is checked, click this item to remove the check. Then click **Transf|Translation**, type *AB* into the "vector" box, *ABC* into the "vertices" box, and press *Enter*. A new triangle appears, probably named $A_0B_0C_0$. According to the definition of vector translation, points A_0 and *B* have to coincide. Nevertheless, *Wingeom* has assigned a new label to what is really the same point. The program thinks that the points are different — indeed, because A_0 had to be *calculated*, it might actually differ from *B* in the twentieth decimal place! This is a situation when you might feel obliged to override the program's lack of understanding, and you *can*: Click **Edit|Undo** (or press *Ctrl+Z*) to undo the translation, click **Transf|Save labels** to turn this feature back on, then click **Edit|Undo undo** (or press *Ctrl+Y*) to redo the translation. Notice that point A_0 does not appear this time. The program has been given permission to regard the nearness of A_0 and *B* as sufficient reason to *identify* these points in its records. This feature only applies to new points generated by the **Transf** menu, by the way.

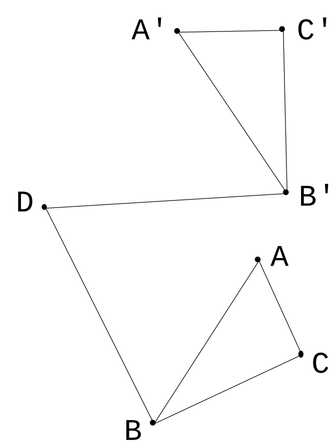
34. Click **File|New** to begin a new figure. Click **Units|Random|Right triangle** (notice the word *right*). Even though the vertices of this figure are randomly generated, they can *not* be dragged independently of each other. Try it — the triangle moves rigidly as a unit. This is because the program has been taught to respect the right-angle definition.

35. Put the mouse into **Btns|Segments** mode and right-click a random point *D* onto the screen, somewhere outside triangle *ABC*. Click **Transf|Rotation**, type *D* into the *center* box, notice

that the "dilation factor" is 1.0 (meaning no dilation), type *ABC* into the "vertices" box, and type 90# into the "angle" box (you must include the symbol #). Press *Enter* to see the result of rotating triangle *ABC* around the pivot point *D*. The size of the rotation angle depends on the value of the number #. Before discovering its value in the next paragraph, estimate it by examining your figure.

36. Click **Animate|# slider**. The resulting dialog box (which can be left open indefinitely) displays the current value of #, and gives you a few ways to change that value. One method is to type a new value for the highlighted text, so type 0.5 into the edit box and press *Enter*. Notice that the figure changes when the value of # does: You now see the result of applying a 45-degree rotation, centered at *D*, to triangle *ABC*. Point at the arrow at one end of the scroll bar, press the left button, and hold it down. The control will slide slowly, the value of # changing as a result. As # changes, so does the size 90# of the rotation angle. You can also slide the control directly by dragging it. The simplest thing is to click **autoreverse**, which moves the bar automatically back and forth. Notice that the dialog box disappears, and that the caption on the drawing window tells you that you must press the *Q* key if you want this animation to stop. Watch for a while, then press *Q*.

37. Click **Line|Segments**, type *BDB'* into the edit box, and press *Enter*. The segments *BD* and *DB'* appear. Click **Edit|Highlights|Line attributes**, type *BDB'* into the edit box, click the **style** button until "dotted" appears, and press *Enter*. Press *Escape* to close this dialog box. Your figure should now resemble the illustration.



38. Click open the **Meas** dialog. Type # and press *Enter*. Type 90# and press *Enter*. Type <*BDB'* and press *Enter*. Close the dialog. Explain the coincidence of displayed values. Use the #-slider to vary the value of #, and notice the changing screen display.

39. Click the **Transf|Save labels** item to enable it. Click **Transf|Rotation**, type *D* into the "center" box, $A'B'C'$ into the "vertices" box, 90# into the "angle size" box, then press *Enter*. Triangle $A''B''C''$ appears. Press *F7* four times. There should now be seven triangles on the screen, the last being $A_3''B_3''C_3''$.

40. Type 2/3 into the edit box that displays the current value of #, and press *Enter*. Explain why there are now only six triangles on the screen. Where did $A_3''B_3''C_3''$ go? If you need a hint, type 0.6 into the edit box and press *Enter*.

41. To change the number of decimal places in displayed calculations, click **Edit|Decimals**, type a nonnegative integer less than 19, and press *Enter*. The new format will become apparent as soon as any numerical display is refreshed. For example, you could use the mouse to drag the original triangle *ABC* around the screen.

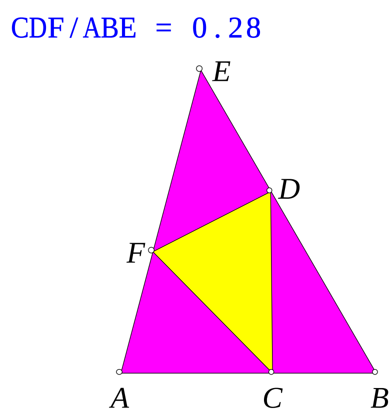
42. Click **File|New** to begin a new figure. For a very simple start, click **Units|Segment**, and press *Enter*. A segment of unit length (this refers to the scale on the underlying graph paper) appears. Click **Line|Angles|New**, type *AB* into the “initial ray” box, 75 into the “angle size” box, and press *Enter*. Ray *AC* appears. Now type *BA* into the “initial ray” box and –60 into the “angle size” box, and press *Enter* again. Ray *BD* appears. Notice the use of a negative sign to describe the second angle. What positive size would have produced the same ray *BD*?

43. The intersection of these rays may well be outside the drawing window. One way to label this (perhaps invisible) point is to click **Point|Intersection|Line-line**, type *AC* and *BD* into the boxes, and press *Enter*. The intersection *E* should now be visible.

44. Triangle *ABE* has been constructed so that its angles are 75, 60, and 45 degrees. To finish the job, click **Edit|Delete|Point**, type *CD* into the box, and press *Enter*. Click **Line|Extensions**, type the list *AE*,*BE* into the box, and press *Enter*.

45. Type 0.6 into the edit box that displays the current value of #, and press *Enter*. Then click the **Point|on Segment** item. This way of marking points on segments offers unlimited possiblilities for animation and control. For example, type the list *AB*,*BE*,*EA* into the “relative to segment” box. If you were to click **mark** now, with 0.5 in the “coordinate” box, you would see all three midpoints of the sides of triangle *ABE* appear. Instead, replace the 0.5 by the symbol #, which you know stands for the value 0.6, and press *Enter*. Point *C* appears on segment *AB*, 60% of the way from *A* to *B*, point *D* appears on segment *BE*, 60% of the way from *B* to *E*, and point *F* appears on segment *EA*, 60% of the way from *E* to *A*.

46. Put the mouse into **Btns|Segments** mode and use the left button to connect the vertices of triangle *CDF*. Notice that the program has reused the discarded labels *C* and *D*, by the way. Click **Edit|Highlights|Fill**, type *ABE* into the “polygon” box, click a red cell in the fill-color window, and click **fill**. The large triangle now has a red interior. Type *CDF* into the edit box, click the yellow cell, and click **fill** again. Notice that it was necessary to color the large triangle *first*. Close the dialog box. The figure should look like the illustration.



47. Click open the **Meas** dialog, ask for the ratio of areas *CDF*/*ABE* (remember that it is all right to use lower case), and close the dialog. The display should show that the area of triangle *CDF* is 28% of the area of triangle *ABE*.

48. Predict the value of the area ratio when *C*, *D*, and *F* are the midpoints of their respective sides. Set # equal to 0.5 to see whether you were right. Can *CDF* be exactly 50% of *ABE*?

49. Make the drawing window the active window (click its title, for example), press *Ctrl+L* to turn off the labels, press *Ctrl+D* a couple of times to turn off the circles, then click **Autoreverse** in the parameter dialog. Remember to press the *Q* key to stop the animation.

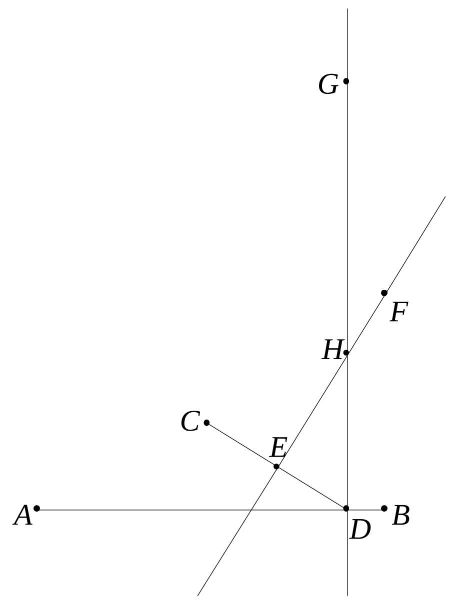
Parabola Lab

1. To construct a parabola, you need a directrix and a focus. Put the mouse into **Btns|Segments** mode and right-click three points onto the drawing surface, with *A* near the bottom on the left, *B* near the bottom on the right, and *C* near the center of the window. Exact placement is not important, for adjustments will be made later. Now use the left button to connect *A* to *B*: Point at either vertex, hold down the left button, drag the pointer to the other vertex, and release. Segment *AB* appears.

2. Right-click a random point *D* onto segment *AB*. To check that it really *is* on the segment, put the mouse into **Btns|Drag vertices** mode and try to drag *D*. It should only slide along *AB*.

3. Put the mouse back into **Btns|Segments** mode and use the left button to connect *C* to *D*.

4. To draw the perpendicular bisector of segment *CD*, click **Line|Perpendiculars|Bisector**, type *CD* into the edit box, and press *Enter* (or click **ok**). The program first marks *E* at the midpoint of *CD*, then draws perpendicular line *EF*. (For inventory purposes, the program needs at least two points on every line, so it introduces *F* as well.) All the points on line *EF* have a special property — what is it?



5. Click **Line|Perpendiculars|General** to draw the line that is perpendicular at *D* to segment *AB*. Type *AB* into the "perpendicular to" box (it might already be there) and *D* into the "through point" box. Click **draw** to see the line *DG*. The view frame will probably be repositioned because of these new points. Press *Escape* to close the dialog box.

6. The simplest way to label the intersection *H* of lines *DG* and *EF* is to point at it and click the right button. (If the intersection does not lie within the window, this method will not work, however. Another way is to click **Point|Intersection|Line-line**, type *DG* and *EF* into the two edit boxes, and press *Enter*.) The figure should now resemble the illustration.

7. Put the mouse into **Btns|Drag vertices** mode. Then use the left button to drag *D* back and forth along segment *AB*. If the menu item **Other|Autoextend** does not have a check mark, give it one by clicking it. This feature enables you to slide *D* past the ends of segment *AB*.

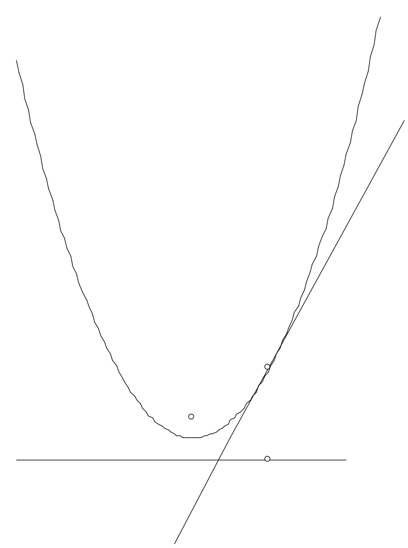
8. No matter what position *D* has on line *AB*, it is always the point on *AB* that is ____________ to *H*. No matter what position *D* has on line *AB*, what can be said about the distances from *H* to the focal point *C* and from *H* to the directrix *AB*?

9. Now that point *H* has been constructed, points *F* and *G* are no longer needed, so click **Edit|Delete|Point**, type *FG* into the edit box, and press *Enter*. Lines *DH* and *CD* have also served their purposes, so click **Edit|Delete|Line**, type the list *DH*,*CD* into the edit box, and press *Enter*. Click **View|Window** (or press *Ctrl+W*) to reposition the figure in the window.

10. As you slide *D* along *AB*, try to visualize the path that *H* is following. Here is an easy way of making this path appear: Click **Animate|Temporary trace**, type *H* into the edit box, and press *Enter*. When you slide *D* along *AB* now, the program retains all of the corresponding positions for *H* in the figure. It is better to move *D* slowly. This is only a temporary display (it disappears as soon as the screen is refreshed).

11. What would the path look like if *C* were closer to *AB*? What would the path look like if *C* were further from *AB*? To explore such questions, first drag *C* to a new position and then regenerate the path by sliding *D* along *AB* again.

12. For a smooth, permanent tracing, click **Animate|Tracing**, then click **new** in the dialog box. Click the **control vertex** button and type *D* into the adjacent edit box. This tells the program that the tracing is generated by dragging vertex *D*. You can leave the default values for “steps” (100), “low” (0.0), and “high” (1.0) as they are. Click the **pen on vertex** button, type *H* into the edit box, and press *Enter* (or click **ok**). The program plots *H* for 100 positions of *D*, and connects the dots.



13. Click **Animate|Temporary trace**, empty the list in the edit box, and press *Enter*. This de-activates the temporary trace path.

14. Press *Ctrl+L* to turn off the labels. They are no longer needed. Press *Ctrl+D* until all the vertices disappear. To highlight specific vertices, open the dialog box **Edit|Labels|Individual**. Type *CDH* into the edit box, uncheck the **show label** box, and click the **circle** button. When you now click **apply**, the three vertices *C*, *D*, and *H* should become visible, each identified by a circle, as in the diagram. Close the dialog box.

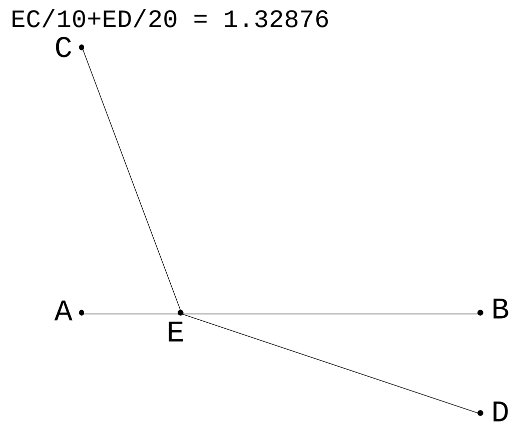
15. The perpendicular bisector *EH* has been left in the figure for a reason — it bears a special relationship to the parabola traced by *H*. As you slide *D* along *AB*, think of some words to describe this relationship: ____

16. If you want the tracing to respond immediately when the vertices of the diagram are moved (*C*, for example), check the menu item **Animate|Monitor tracings**. If this item is not checked, it is necessary to click **Animate|Retrace** to redraw tracings.

Snell's Law Lab

1. Click **Point|Coordinates**. In the dialog box, type 0 into the edit boxes for *x* and *y*. Press *Enter* (or click **mark**) to put *A* = (0,0) onto the screen. Type 12 into the *x* box and press *Enter* again to make *B* = (12, 0) appear. In the same way, mark points *C* = (0,8) and *D* = (12,–3). Press *Escape* to close the dialog box. Click **View|Window** (or press *Ctrl+W*) to center the figure within the window. Press *Ctrl+A* to remove the axes.

2. Check that the mouse is in **Btns|Segments** mode, then use the left button to draw the segment *AB*. Use the right button to click a point *E* onto segment *AB*. Then use the left button to draw segments *EC* and *ED*.



3. Uncheck the item **Other|Measurment|Units**. Open the **Meas** dialog box and put the sum *EC*/10 + *ED*/20 on display. Press *Escape* to close the dialog box. The figure represents an interface *AB* between two media, and *CED* is a path followed by an object whose speed is 10 in the upper medium and 20 in the lower medium. The displayed measurement is the total time needed to traverse the path.

4. Put the mouse into **Btns|Drag vertices** mode. Use the left button to drag *E* back and forth along segment *AB*, trying to make the displayed measurement as *small* as you can.

5. Once *E* is in its optimal position, open the **Meas** dialog, ask for the length of *AE*, and press *Escape* to close the dialog box. Before moving on, record the two values displayed in the figure: *AE* = _________________ and *EC*/10 + *ED*/20 = _________________ .

6. If the objective had been to make the value of the sum *EC*/10 + *ED*/10 as small as possible, your search would not have led you to the same point *E*. Guess which point on segment *AB* would have been the optimal choice for *E*, then check your guess by doing the search. Begin by using the **Meas** dialog to delete *EC*/10 + *ED*/20 and replace it by *EC*/10 + *ED*/10. Record your findings: *AE* = _________________ and *EC*/10 + *ED*/10 = _________________ .

7. New example: Search for the point *E* that makes *EC*/20 + *ED*/10 as small as possible. Record your findings: *AE* = _________________ and *EC*/20 + *ED*/10 = _________________ . Notice that segment *EC* is longer than it was in either of the two preceding solutions. Explain why this could have been predicted.

8. Click **Line|Segment**, type the list *AC*,*BD* into the edit box, and press *Enter*. Segments *AC* and *BD* should appear. Use the mouse to drag *E* along segment *AB*. As *E* moves from *A* to *B*, what happens to the sizes of angles *ACE* and *BDE*? Are they ever the same size?

9. You have now explored three problems of the same general form: Find the point *E* that minimizes the value of the expression $EC / r_C + ED / r_D$. The expression represents *time*, because the numerators of the fractions represent *distances* and the denominators represent *rates*. A few more examples and measurements will help to reveal a pattern. Fill in the missing entries of the table below, and notice that there are two new quantities of interest. Remember that each row requires a separate search.

r_C	r_D	*EA*	$EC / r_C + ED / r_D$	*EA/EC*	*EB/ED*
10	30				
10	20				
10	10				
40	30				
20	10				
30	10				

10. As the rate r_D decreases, what happens to the optimal position of the point *E*? What happens to the length of segment *ED*? What happens to the size of angle *BDE*? Are r_D and angle *BDE* linearly related?

11. The simple pattern known as *Snell's Law* relates r_C, r_D, *EA/EC*, and *EB/ED*. It can be inferred by examining the table. Express this relationship in words. Express it in symbols.

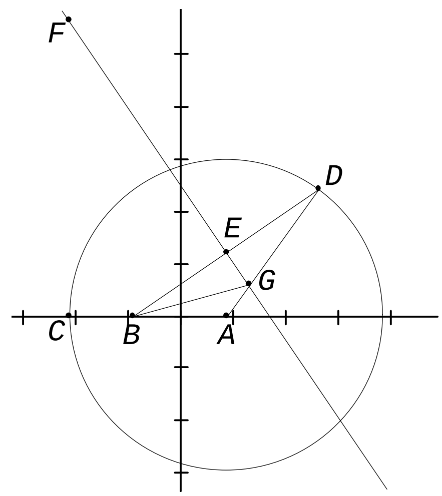
12. Write a trigonometric description of Snell's Law that relates the rates r_C and r_D to the angles *ACE* and *BDE*.

Ellipse Lab — Part I

1. Click **Points|Coordinates**. Type 9 into the *x*-box and 0 into the *y*-box, and press *Enter* (or click **mark**). Point $A = (9,0)$ appears on the screen. In the same way, mark points $B = (–9,0)$ and $C = (–21,0)$. Press *Escape* to close the dialog box.

2. Click **Circle|Radius-center**, type *A* into the “centered at” box, click the **through** button, type *C* into the adjacent box, and click **draw**. You should now see the circle centered at *A* that goes through *C*. Press *Escape* to close the dialog box.

3. With the mouse in **Btns|Segments** mode, point anywhere *on* the circle and *right*-click to mark *D*. Use the left button to draw segments *AD* and *BD*.



4. The point *D* is confined to the circle, but can be slid along it. To see this, put the mouse into **Btns|Drag vertices** mode, point at *D*, press the left button, and hold it down while you slide the mouse.

5. Click **Line|Perpendiculars|Bisector**, type *BD* into the edit box, and press *Enter*. Notice that the program marked midpoint *E* on segment *BD* when it drew the bisector *EF*.

6. Put the mouse into **Btns|Segments** mode. Right-click the point where line *EF* intersects segment *AD*. The intersection point should now be labeled *G*.

7. Use the left button to connect *G* to *B*. The figure should now resemble the illustration.

8. As you answer the following questions, you may wish to move *D* around the circle (first put the mouse back into **Btns|Drag vertices** mode) and observe what happens to the constructions, especially point *G*. The **Meas** dialog box may also be useful.

   (a) How do the lengths *GB* and *GD* compare?

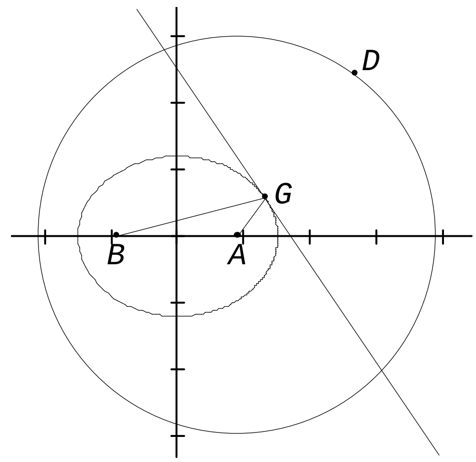
   (b) What must always be true about the sum of the distances *GB* and *GA*, no matter what the position of point *D*? Explain.

   (c) Describe what the trace of point *G* will look like as point *D* moves around the circle.

Ellipse Lab — Part I (continued)

9. To see a picture of the path of *G*, open the inventory dialog box **Animate|Tracing** and click **new**. Click the **control vertex** button and type *D* into the adjacent box (this tells the program to produce the tracing by sliding *D*). Click the **pen on vertex** button and type *G* into the adjacent box. Use the default values for “steps” (100), “low” (0.0), and “high” (1.0). Press *Enter*. The program will plot 100 different positions for *G*, and connect the dots to form a curve. The construction is superimposed.

10. It is time to clean up a bit. Click **Edit|Delete|Line**, type *DB*,*DA* into the box, and press *Enter*. Click **Line|Segment**, type *AG* into the box, and press *Enter*. Click **Edit|Labels|Individual**, click the **show label** box to uncheck it, and click apply. Then type *ABDG* into the “vertices” box, click the **circle** button, and click **apply**. Press *Escape* to close the dialog. Your figure should now resemble the illustration.



11. Click **Meas**, type *G* into the edit box, and press *Enter*. The coordinates of point *G* are added to the figure. Close this dialog box (press *Escape*) and return to sliding *D* around the circle. This makes *G* slide along a curve known as an *ellipse*. The path of *G* intersects the coordinate axes at four lattice points. Find these coordinates exactly. Is it a coincidence that the distance between the *x*-intercepts is equal to the radius of the circle? Explain.

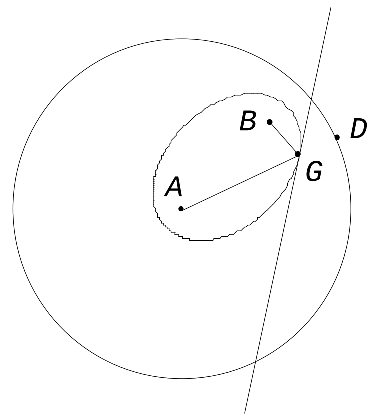
12. Summarize the construction roles played by point *A*, point *D*, radius *AC*, and line *EF*.

13. It so happens that the focal radius *GA* is always three fifths of the distance from *G* to the vertical line $x = 25$. Here is one way to confirm this fact: Use the **Point|Coordinates** dialog to plot two points on the line, say $H = (25,–30)$ and $I = (25,30)$. Draw the line by clicking **Line|Lines**, typing *HI* into the edit box and pressing *Enter*. Then mark the point *J* on *HI* that is closest to *G* by clicking **Line|Perpendiculars|Altitude**, typing *HI* into the “perp to” box, *G* into the “from point” box, and pressing *Enter*. Now open the **Meas** dialog and ask for the values of *GA*, *GJ*, and *GA/GJ*, one at a time. Close this dialog and return to sliding *D* around the circle, keeping an eye on the displayed values. Notice that the ratio *GA/GJ* does indeed have a constant value, even though *GA* and *GJ* are *not* constant.

14. The line $x = 25$ is called a *directrix*. An ellipse has two of them. Can you find an equation for the other directrix of this ellipse?

15. The constant ratio *GA/GJ* (3/5 in this example) is called the *eccentricity* of the ellipse. Verify that the ratio *AB*/*AD* is also 3/5. This is not a coincidence.

Ellipse Lab — Part II



16. Click **File|New** to start a new sketch. Instead of plotting three specific points as in part I, put the mouse into **Btns|Circles** mode, and use the right button to mark three random points *A*, *B*, and *C*, with *B* closer to *A* than *C* is. Left-click *A*, drag the mouse to *C*, and release. This creates the circle in step 2 of part I. (If *B* is not inside the circle, drag it inside before proceeding.) Put the mouse into **Btns|Segments** mode and continue with the rest of the construction of *G*.

    Unlike the specific points in part I, these points *A*, *B*, and *C* can be dragged around the screen. It is interesting to study the effect that different positions have on the appearance of the elliptical path of point *G*.

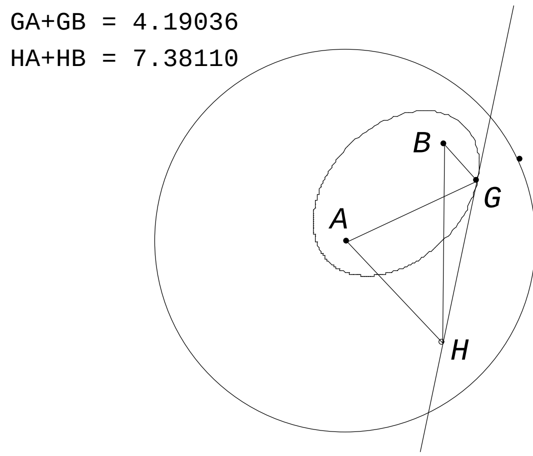
17. Click **Animate|Tracing** and repeat step 9 of part I. For a more accurate tracing, put 300 into the "steps" box. Check the menu item **Animate|Monitor tracings**, so that this tracing will respond when you drag points other than *D*. **File|Save** the figure before proceeding. Drag *B* close to the center *A*. Notice that the shape of the ellipse is different this time. Now drag *B* close to the circle and notice the new shape. You can also experiment with changing the size of the circle, by dragging *C*. Notice that the construction falls apart if *B* is outside the circle. Keep in mind that the sum *GA*+*GB* always equals the _______________ of the circle. The sum *GA*+*GB* is equal to what dimension of the ellipse itself? (see step 11 of part I)

18. The eccentricity *AB*/*AD* describes the shape of the ellipse. In the part I example, the eccentricity was 3/5. Use the **Meas** dialog to display the value of *AB*/*AD*. When you move points *A*, *B*, or *C*, the eccentricity will change. What is the *range of values* of the eccentricity? What words would you use to describe the shape of the ellipse when the eccentricity has (a) a value near zero? (b) a value near one?

19. A Challenge**:** In part I, the lines $x = 25$ and $x = -25$ were the directrices for the ellipse. Recall that, for *any* point *G* on that ellipse, the distance from *G* to the focus *A* was three fifths of the distance from *G* to the corresponding directrix. This should help you construct the directrices for the generic ellipse in your figure. These lines should of course change whenever you change the ellipse by moving *A*, *B*, or *C*.

Ellipse Lab — Part III

20. Use **File|Old** to retrieve the figure saved in step 17 above. With the mouse in **Btns|Segment** mode, right-click a new point *H* onto the line *EF* (the only line visible in the figure). Use the left button to draw segments *HA* and *HB*. Click open the **Meas** dialog, ask for the values of *GA*+*GB* and *HA*+*HB* (one at a time), then *Escape* from the dialog.



21. Put the mouse back into **Btns|Drag vertices** mode and slide *D* around the circle, thereby driving *G* around the ellipse. The displayed measurements should confirm that *GA*+*GB* is constant for all points *G* on the ellipse. This constant is equal to the ___________ of the circle. Now slide *H* along the line *EF*. Notice that *HA*+*HB*, which is evidently not constant, is never ___________ than the sum *GA*+*GB*. What does this tell us about the relationship between the ellipse and the line?

22. Press *Ctrl*+*L* to turn on all the vertex labels. Slide point *H* so that *G* is between *F*and *H*, then **measure** the angles *FGB* and *HGA*. Slide *D* around the circle so that *G* slides around the ellipse, and watch the angle measurements. What important property of the ellipse does this illustrate? Explain how this property ensures that the minimum value of *HA*+*HB* for a point *H* on the line *FG* must indeed occur when *H* coincides with *G*.

Ellipse Lab — Part IV (summary)

23. Click **File|New** to make a fresh start. Put the mouse in **Btns|Segments** mode, right-click three points *A*, *B*, and *C* onto the screen, and use the left button to draw segments *AC* and *BC*.

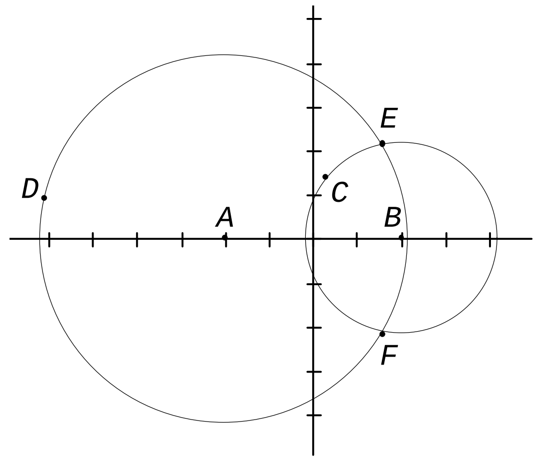
24. Suppose that points *A* and *B* represent the focal points of an ellipse, and that *C* is a point on that ellipse. The problem is to construct the line that is tangent to the ellipse at *C*. This can be done in just two steps. First click **Line|Angles|Bisect**, type *ACB* into the edit box, and press *Enter*. The bisecting ray *CD* should appear. Next click **Line|Perpendiculars|General**, type *CD* into the "perpendicular to" box, type *C* into the "through point" box, and click **draw**. Tangent line *CE* appears. *Escape* from the dialog.

25. Because there is no longer a need for the bisecting ray, click **Edit|Delete|Line** to remove *CD*, and click **Edit|Delete|Point** to remove point *D*. Now put the mouse into **Btns|Drag vertices** mode. Drag *C* around the screen and notice how the line *CE* reacts. Because the sum of lengths *AC*+*BC* is not kept constant, each line *CE* is tangent to a different ellipse. To see the ellipses, open the **Units|Conic** dialog box. Click the **ellipse** button, type *AB* into the "directrix" box, type *C* into the "point on" box, and click **draw**. The ellipse is added to the conic inventory, and the curve appears on the screen. *Escape* from the dialog.

Hyperbola Lab — Part I

1. Open the **Point|Coordinates** dialog box. Type –2 into the "*x*" box and 0 into the "*y*" box, then press *Enter* to make *A* = (–2,0) appear on the screen. In a similar fashion make *B* = (2,0) appear. Press *Escape* to close the dialog box.

2. With the mouse in **Btns|Segments** mode, right-click a random point *C* onto the screen approximately midway between *A* and *B*.



3. Click **Circle|Radius-center**, type *B* into the "centered at" box, click the **through** button, type *C* into the adjacent box, and click **draw**. Now type *A* into the "centered at" box, click the **radius** button, type 2+*BC* into the adjacent box, and click **draw**. *Escape* from the dialog box.

4. Unless *C* is too close to *B*, the circles you just drew should intersect in two places. Right-click either point. The labels *E* and *F* will be applied to the intersections.

5. The coordinates of *E* and *F* can be displayed: Open the **Meas** dialog box, type *F* and press *Enter*, type *G* and press *Enter*, then press *Escape*.

6. Put the mouse into **Btns|Drag points** mode. Drag point *C* (which is actually the only point that *can* be dragged). You will see the coordinates for many points that are 2 units closer to *B* than they are to *A*. The *y*-coordinate of one of these intersection points is 5; its *x*-coordinate is approximately _____________. Another special case occurs when the intersection points *merge*; the resulting *x*-coordinate is _____________.

7. Click **Animate|Temporary trace**, type *EF* into the edit box, and press *Enter*. When you drag *C* around the screen now, the program accumulates all of the corresponding positions for *E* and *F* in the figure. More points are plotted when you move *C* slowly. This is only a temporary display, however — it disappears as soon as the screen is refreshed. A more permanent display is obtained next. The curve is called a *hyperbola*.

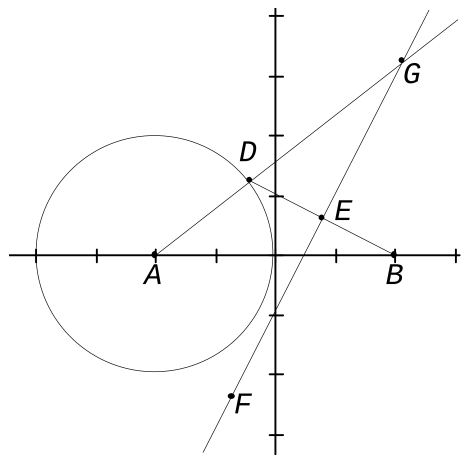
Hyperbola Lab — Part II

8. Begin a fresh sketch with the same points *A* and *B*. One way to do this is to click **Edit|Undo** (or press *Ctrl+Z*) until *C* disappears. Another is to click **File|New** and re-enter the coordinates for *A* = (–2, 0) and *B* = (2, 0) a second time, using the **Point|Coordinates** dialog box.

9. Open the **Circle|Radius-center** dialog box. Type *A* into the “centered at” box, click the **radius** button, type 2 into the adjacent box, and click **draw**. A circle of radius 2 appears, with a random point *C* marked on it. Press *Escape* to close the dialog box.

10. With the mouse in **Btns|Segments** mode, right-click a random point *D* onto the circle, fairly close to the *y*-axis. Use the left button to draw the segment *BD*. With the mouse in **Btns|Rays** mode, use the left button to draw the ray *AD*.

11. Click **Line|Perpendiculars|Bisector**, type *BD* into the edit box, and press *Enter*. Notice that the program automatically marks the midpoint *E* of segment *BD* when it draws the bisector.



12. Point at the intersection of ray *AB* and line *FE*, and right-click. You should see label *G* appear, as in the illustration.

13. Explain why *GA* – *GB* = *AD* = 2, no matter what the position of *D*.

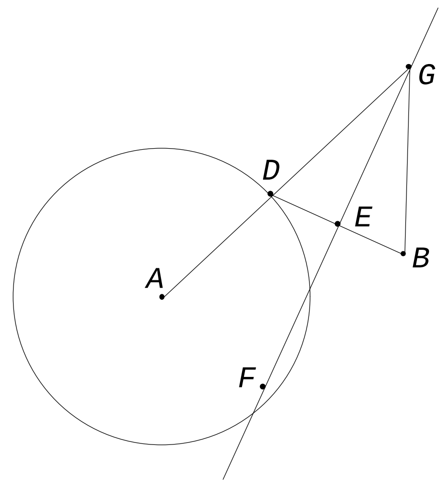
14. Click **Animate|Temporary trace**, type *G* into the edit box, and press *Enter*. Put the mouse into **Btns|Drag vertices** mode and drag *D* slowly around the circle. Explain why the trace of *G* consists of points that already appeared in step 7 above. Notice also that, for many positions of *D*, there is no point *G*.

15. For a smooth, permanent tracing, open the **Animate|Tracing** dialog box and click the **new** button. Click the **control vertex** button and type *D* into the adjacent box (this tells the program to produce the tracing by sliding *D*). Click the **pen on vertex** button and type *G* into the adjacent box. Use the default values for “steps” (100), “low” (0.0), and “high” (1.0). Press *Enter*. The program will plot 100 different positions for *G*, and connect the dots to form a curve. The construction is superimposed.

Hyperbola Lab — Part III

18. Click **File|New** to start a new sketch, and put the mouse into **Btns|Segments** mode. If the coordinate axes are still on the screen, press *Ctrl+A* to remove them. Use the right button to mark two random points *A* and *B*, and a third random point *C* that is closer to *A* than *B* is.

19. Click **Circle|Radius-center**, type *A* into the "centered at" box, click the **through** button, type *C* into the adjacent box, and click **draw**. You should now see the circle centered at *A* that goes through *C*. Press *Escape* to close the dialog box.



20. Point anywhere *on* the circle and click the right button to mark *D*. Then use the left button to draw the segments *AD* and *BD*.

21. The point *D* is confined to the circle, but can be slid along it. To see this, put the mouse into **Btns|Drag vertices** mode, point at *D*, press the left button, and hold it down while you slide the mouse. When you are done, leave *D* roughly between *A* and *B*.

22. Click **Line|Perpendiculars|Bisector**, type *BD* into the edit box, and press *Enter*. Notice that the program marked midpoint *E* on segment *BD* when it drew the bisector *EF*.

23. So that the next label will appear, check the item **Other|Autoextend** by clicking it. Click **Point|Intersection|Line-line**, type *EF* into one box and *AD* into the other, and click **mark**. Label *G* should now mark the intersection of line *EF* and ray *AD*. Press *Escape* to close the dialog box.

24. Put the mouse into **Btns|Segments** mode, and use the left button to draw *BG*. The figure should now resemble the illustration.

25. As you answer the following questions, you may wish to move *D* around the circle (first put the mouse back into **Btns|Drag vertices** mode). Observe what happens to the constructions, especially point *G*. The **Meas** dialog box may also be useful.

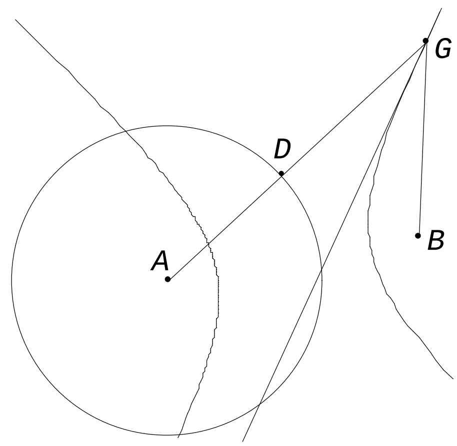
    (a) How do the lengths *GB* and *GD* compare?

    (b) What must always be true about the difference *GA* – *GB*, no matter what the position of point *D*? Respond carefully — this difference is not always positive.

    (c) Describe what the trace of point *G* will look like as point *D* moves around the circle.

Hyperbola Lab — Part III (continued)

26. To see a picture of the path of *G*, open the inventory dialog box **Animate|Tracing** and click **new**. Click the **control vertex** button and type *D* into the adjacent box (this tells the program to produce the tracing by sliding *D*). Click the **pen on vertex** button and type *G* into the adjacent box. Use the default values for "steps" (100), "low" (0.0), and "high" (1.0). Press *Enter*. The program will plot 100 different positions for *G*, and connect the dots to form a curve called a hyperbola. The construction is superimposed.



27. It is time to clean up a bit. Click **Edit|Delete|Line**, type *DB* into the box, and press *Enter*. Click **Edit|Labels|Individual**, click the **show label** box to uncheck it, type *EF* into the "vertices" box, and click **apply**. Press *Escape* to close the dialog. Press *Escape* to close this dialog. Your figure should now resemble the illustration.

28. With the mouse in **Btns|Drag points** mode, you can drag *A*, *B*, and *C* around the screen and see the effect that this has on the tracing. Check the menu item **Animate|Monitor tracings**, so that this tracing will respond when you drag points other than *D*. **File|Save** the figure before proceeding. It is interesting to see what happens to the trace when *B* is closer to *A* than *C* is — the hyperbola turns into an ellipse!

29. Summarize the construction roles played by point *A*, point *D*, radius *AC*, and line *EF*.

Hyperbola Lab — Part IV (summary)

30. Click **File|New** to make a fresh start. Put the mouse in **Btns|Segments** mode, right-click three points *A*, *B*, and *C* onto the screen, and use the left button to draw segments *AC* and *BC*.

31. Suppose that points *A* and *B* represent the focal points of a hyperbola, and that *C* is a point on that hyperbola. The problem is to construct the line that is tangent to the hyperbola at *C*. This can be done very quickly. First click **Line|Angles|Bisect old**, type *ACB* into the edit box, and press *Enter*. Bisecting ray *CD* should appear. To make it into a tangent line, click **Line|Extensions**, type *DC* into the edit box, and press *Enter*.

32. Now put the mouse into **Btns|Drag vertices** mode. Drag *C* around the screen and notice how the line *CD* reacts. Because the difference of lengths *AC*–*BC* is not kept constant, each line *CD* is tangent to a different hyperbola. To see the hyperbolas, open the **Units|Conic** dialog box. Click the **hyperbola** button, type *AB* into the "directrix" box, type *C* into the "point on" box, and click **draw**. The hyperbola is added to the conic inventory, and the curve appears on the screen. *Escape* from the dialog.